Mathematical Statistics and Probability Theory

Volume B

Proceedings of the 6th Pannonian Symposium on Mathematical Statistics, Bad Tatzmannsdorf, Austria, September 14-20, 1986

Edited by

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CONTENTS

PREFACE

The past several years have seen the creation and extension of a very conclusive theory of statistics and probability. Many of the research workers who have been concerned with both probability and statistics felt the need for meetings that provide an opportunity for personal contacts among scholars whose fields of specialization cover broad spectra in both statistics and probability: to discuss major open problems and new solutions, and to provide encouragement for further research through the lectures of carefully selected scholars, moreover to introduce to younger colleagues the latest research techniques and thus to stimulate their interest in research.

To meet these goals, the series of Pannonian Symposia on Mathematical Statistics was organized, beginning in the year 1979: the first, second and fourth one in Bad Tatzmannsdorf, Burgenland, Austria, the third and fifth in Visegrád, Hungary. The Sixth Pannonian Symposium was held in Bad Tatzmannsdorf again, in the time between 14 and 20 September 1986, under the auspices of Dr.Heinz FISCHER, Federal Minister of Science and Research, Theodor KERY, President of the State Government of Burgenland, Dr.Franz SAUERZOPF, Vice-President of the State Government of Burgenland and Dr.Josef SCHMIDL, President of the Austrian Statistical Central Office. The members of the Honorary Committee were Pál ERDÖS, Władisław ORLICZ, Pál RÉVÉSZ, Leopold SCHMETTERER and István VINCZE; those of the Organizing Committee were Wilfried GROSSMANN (University of Vienna), Franz KONECNY (University of Agriculture of Vienna) and, as the chairman, Wolfgang WERTZ (Technical University of Vienna).

About 160 scholars from 17 countries participated in this conference, a particularly large number of them came from Hungary, Poland and Germany, but more distant countries were well-represented, too, such as The Netherlands, Spain and Portugalia; moreover there were several participants from the United States of America, Canada, Israel and the Republic of South Africa.

The scientific program of the Sixth Pannonian Symposium on Mathematical Statistics covered more than 100 contributions, most of them in the form of contributed lectures, a few of them in the framework of a poster session. The four specially invited plenary lectures were delivered by Luc DEVROYE (Montréal), Herbert HEYER (Tübingen), Petr MANDL (Praha) and Madan L.PURI (Bloomington). There was a rather broad range of the topics, including probability theory, theory of stochastic processes, the mathematical foundations of statistics, decision theory, statistical methods and some applications.

A selection of the contributions of the conference is published in these proceedings, consisting of two volumes. Whereas this book contains papers emphasizing the development of statistical and probabilistic methods, the other volume, with the subtitle "Theoretical Aspects", includes primarily contributions concerned with the mathematical foundations of statistics and probability theory (a list of the contents of this volume can be found on p.261). It has been the aim of the editors to publish new and significant results; the assistance of numerous referees constituted an indispensible help in approaching this objective - the editors wish to express their deep gratitude to all the referees; they are listed below. Despite of the careful redaction of the volume, the responsibility for the manuscripts remains with the authors.

Roughly speaking, the papers of this volume appertain four main topics: probability and stochastic processes, testing hypotheses, estimation and applications. The *probabilistic* articles have obvious applications in statistics or even explicitely refer to them; to the first group belong the papers by Erdös&Révész (on random walks; several interesting open problems are formulated), Glänzel (characterization theorems), Gyires (linear prediction), Móri (limit theorems for waiting times) and Stadje (stopping of processes); with the second one rank the articles by Athayde&Gomes (extreme value limit theorems applied to testing problems) and by Ignatov&Kaishev (certain distributions applied to contingency tables). Three papers deal with *testing statistical hypotheses*: Bajorski&Ledwina (rank tests), Drost (chi-square type tests) and Prášková&Ratajová (Bayesian analysis of contingency tables, in particular the construction of credible intervals). Various *estimation problems* are considered: density estimation for dependent samples by Deddens&Peligrad&Yang, estimators based on censored data by Ferenstein, Schick&Susarla, parameter estimation after certain Box-Cox-transformations by Rukhin and sequential estimation in stochastic processes by Pruscha; González Manteiga&Vilar Fernández use nonparametric criteria for estimating parameters of time series. *Methods and applications* are dealt with by Banjević&Nedeljković (technical application), Gupta&Liang (selection procedures), Krzyśko&Wachowiak (classification), Malisić (time series models) and Weron&Weron (use of stable distributions in relaxation problems). Mandl surveys certain connections between statistics and control theory.

The organization of the Sixth Pannonian Symposium on Mathematical Statistics was made possible by the valuable help of many institutions and individuals. The organizers take the opportunity to express their thanks, in particular, to the following institutions: the State Government of Burgenland (Departments of Official Statistics, of Affairs of Communes and of Tourist Trade), the Federal Ministry of Science and Research, the Austrian Statistical Society, the Creditanstalt, the Volksbank Oberwart, the Raiffeisenverband Burgenland, the Local Government Bad Tatzmannsdorf, the Kurbad Tatzmannsdorf AG and the Spa Commission of Bad Tatzmannsdorf. The interest of the Authorities in the conference has been emphasized by the attendance of numerous representati-

ves of public life at the opening ceremony of the symposium; the President of the State Government of Burgenland honoured the congress by opening it himself.

Last not least, cordial thanks are due to the ladies who helped in the local organization and in mastering the extensive paperwork and typing.

Bad Tatzmannsdorf,
April 1987

Wolfgang Wertz

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We express our deepest gratitude to the following referees, who gave us indispenable advice for the editorial process. They helped us in the selection of the papers published in the two proceedings volumes on the Sixth Pannonian Symposium on Mathematical Statistics; their constructive criticism and numerous valuable suggestions to the authors lead to a considerable improvement of several manuscripts.

The editors

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MULTIVARIATE EXTREMAL MODELS UNDER NON-CLASSICAL SITUATIONS

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ABSTRACT. The limiting distribution of top order statistics in a non-classical set-up, where the independence structure remains valid, is reviewed in this paper. We essentially place ourselves under Mejzler's hypothesis - independent X_k's with distribution function $F_k(x)$, $k\geq 1$, satisfying the uniformity condition for the maximum. Notice that the results presented are obviously valid not only on Mejzler's $\mathbf{M}_1$ class, but also on refinements $\mathbf{M}_r$, $r>1$, of Mejzler's class and in $\mathbf{M}_\infty = \bigcap_{r\geq 1} \mathbf{M}_r$, a non-trivial extension of the class S of max-stable distributions. Generalizing the multivariate GEV model, other multivariate extremal models based on functions H(x) belonging to $\mathbf{M}_1$(or to $\mathbf{M}_\infty$) are introduced and inference techniques are developed for a multivariate extremal Pareto model.

1. INTRODUCTION AND PRELIMINARIES

For sequences of independent, identically distributed (i.i.d.) random variables (r.v.'s) the non-degenerate limiting structure, whenever it exists, of the normalized top i order statistics (o.s.), i a fixed integer, is well-known and characterized by the joint probability density function (p.d.f.)

$$h(z_1,\ldots,z_i) = g(z_i) \prod_{j=1}^{i-1} \{g(z_j)/G(z_j)\}, z_1>\ldots>z_i \tag{1}$$

where $G(z) = G_\theta(z)$ is in the class S of max-stable distribution functions (d.f.'s), often called Generalized Extreme Value (GEV) d.f.'s, i.e.

$$G_\theta(z) = \begin{cases} \exp(-(1-\theta z)^{1/\theta}),\ 1-\theta z>0,\ z\in \mathbb{R} & \text{if } \theta\neq 0 \\ \\ \exp(-\exp(-z)),\ z\in \mathbb{R} & \text{if } \theta=0 \end{cases} \tag{2}$$

P. Bauer et al. (eds.), Mathematical Statistics and Probability Theory, Vol. B, 1–9.

$g_\theta(z) = \partial G_\theta(z)/\partial z$.

If we drop the hypothesis of identical distribution — a more common set-up in applications — , and deal with sequences $\{Y_n\}_{n\geq 1}$ of r.v.'s whose associated sequence of partial maxima $\{M_n^{(1)} = \max_{1\leq j\leq n} Y_j\}_{n\geq 1}$, suitably normalized, converges weakly, as $n\to\infty$, to a r.v. in Mejzler's class $\mathbf{M}_1$ [Mejzler, 1956], the limiting structure of the top i o.s., detailed in section 2, is still a multivariate extremal vector with p.d.f. given by (1), but with $G\in\mathbf{M}_1$. Notice that this is just a corollary of the result of Weissman (1975), expressed here in a slightly different context. If we work instead with refinements $\mathbf{M}_r$, $r>1$, of Mejzler's class $\mathbf{M}_1$, or with $\mathbf{M}_\infty = \bigcap_{r\geq 1}\mathbf{M}_r$ [Graça Martins and Pestana, 1985], analogous results are obtained. Notice that $\mathbf{M}_\infty$ is the smallest class containing the class S of max-stable d.f.'s (2) that is closed under pointwise products and limits. This class seems thus to provide a very general framework for the study of sample maxima, justifying several models put forward by statistical users [Gomes and Pestana, 1985], the same happening to its multivariate generalizations presented here.

This limiting result is thus the probabilistic background for the introduction of a multivariate extremal $\mathbf{M}_1$ model to analyse the set of the largest observations available, in order to infer tail properties. This model is obviously more general and more realistic than the multivariate extremal GEV model, introduced first, in a slightly different context, by Pickands (1975) and worked out by several authors [Weissman, 1978; Smith, 1984]. Mainly for climatological data, where the i.d. hypothesis fails, a multivariate extremal $\mathbf{M}_1$ model, or at least a multivariate extremal $\mathbf{M}_\infty$ model, has often to be called for.

Among the members of the class $\mathbf{M}_1$ we consider, in sections 3 and 4, the Pareto d.f.'s

$$H_\theta(z) = \begin{cases} (1+\theta z)^{1/\theta},\ 1+\theta z>0,\ z<0 & \text{if } \theta\neq 0 \\ \exp(z),\ z<0 & \text{if } \theta=0 \end{cases} \tag{3}$$

and the multivariate extremal model $(X_1,X_2,\ldots,X_{m+1})$, with p.d.f. given by (1), i replaced by m+1 and G(z) replaced by $H_\theta((x-\lambda)/\delta)$, $\lambda\in\mathbb{R}$ and $\delta\in\mathbb{R}^+$ unknown location and scale parameters respectively to be estimated from the sample. We shall call it a multivariate extremal H_θ model (notice that $H_\theta\in\mathbf{M}_\infty$ if $\theta\geq 0$). Since inference techniques in this model are more easily developed for the particular case $\theta=0$, we shall deal here with discrimination among these models, with $\theta=0$ playing a central and eminent role. With this statistical choice problem in mind, we shall deal with Gumbel statistic

$$G_{m+1} = \{X_1 - X_{r_m}\}/\{X_{r_m} - X_{m+1}\},\quad r_m = [(m+1)/2]+1 \tag{4}$$

for testing $H_0: \theta=0$, in a multivariate extremal H_θ model, versus one-sided or two-sided alternatives, [y] denoting, as usual, the greatest integer smaller than y. This statistic, although suggested to us by means of heuristic reasons turned out to be powerful for testing $H_0:\theta=0$ in a univariate GEV(θ) model [van Montfort and Gomes, 1985, and references therein] and in a multivariate GEV(θ) model [Gomes and Alpuim,1986]. In this same context, an analogue of the Locally Most Powerful (LMP) test statistic

$$\tilde{L}_{m+1} = \sum_{j=1}^{m} \{X_1-X_j\}/\{X_1-X_{m+1}\} \tag{5}$$

is also considered.

In section 3 of this paper we derive (5) and distributional properties of G_{m+1} and $\tilde{L}_{m+1}$, under the validity of the null hypothesis $H_0:\theta=0$. Notice that, although the limiting result, in section 2, is obtained for i fixed, in applications, the model remains valid for reasonably large values of sample size, and asymptotic properties of statistics related to the model may thus be called for.

Finally, in section 4, we compare the power functions of the tests based on the statistics (4) and (5).

2. LIMITING STRUCTURE OF TOP ORDER STATISTICS IN A NON-CLASSICAL SET-UP

Let $\{Y_n\}_{n\geq 1}$ denote a sequence of independent r.v.'s and let $F_j(.)$ be the d.f. of Y_j, $j\geq 1$. Let us denote by $M_n^{(k)}$ the k-th maximum value of $\{Y_1,\ldots,Y_n\}$, $1\leq k\leq n$, and assume further the validity of the uniformity condition for the maximum [Mejzler, 1956], and for a suitable sequence $\{u_n\}_{n\geq 1}$ of real numbers, i.e.

$$\lim_{n\to\infty} \min_{1\leq j\leq n} F_j(u_n) = 1 \tag{6}$$

The limiting structure of $(M_n^{(1)},\ldots,M_n^{(i)})$, i a fixed integer, may thus be derived, analogously to what has been done in an i.i.d. set-up [Leadbetter, Lindgrèn and Rootzèn, 1983], from the limiting structure of $M_n^{(1)}$, obtained by Mejzler.

As a direct corollary of the results on convergence of Bernoulli point processes [Serfozo, 1986] to Poisson point processes, as the probabilities of success are small, we obtain

Theorem 1. Under the conditions stated before, assume that $u_n^{(j)}=u_n^{(j)}(\tau_j)$, are such that

$$1-\frac{1}{n}\sum_{k=1}^{n} F_k(u_n^{(j)}(\tau_j)) = \tau_j/n + o(\tfrac{1}{n}), \text{as } n\to\infty, 1\leq j\leq i, \tag{7}$$

that condition (6) for $u_n^{(j)}(\tau_j)$, $1\leq j\leq i$, $\tau_1<\tau_2<\ldots<\tau_i$ is valid, and let $S_n^{(j)}$ be the number of exceedances of $u_n^{(j)}$ by $\{Y_1,\ldots,Y_n\}$, $1\leq j\leq i$, $n\geq 1$.

Then

$$\lim_{n\to\infty} P[\bigcap_{j=1}^{i}\{S_n^{(j)}=k_j\}] = \begin{cases} e^{-\tau_i}\dfrac{\tau_1^{k_1}}{k_1!}\prod_{j=1}^{i-1}\dfrac{(\tau_{j+1}-\tau_j)^{k_{j+1}-k_j}}{(k_{j+1}-k_j)!} & \text{if } k_1\le\ldots\le k_i \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

i.e., the r.v.'s $S_n^{(j-1,j)} = \#\{k: 1\le k\le n,\ u_n^{(j)}<Y_k\le u_n^{(j-1)}\}$, $1\le j\le i$, $u_n^{(0)}=+\infty$, are asymptotically independent Poisson r.v.'s with mean value $\tau_j-\tau_{j-1}$, $1\le j\le i$, $\tau_0=0$.

Notice that different versions of this result are used by Weissman (1975) and Hall (1978). Indeed our theorem 1 corresponds to a re-statement of theorem 1 of Weissman (1975), with t=1, assuming additionally the uniformity condition (6), which is there derived from a broader hypothesis on extremal processes.

Since

$$P[\bigcap_{j=1}^{i}\{M_n^{(j)}\le u_n^{(j)}\}] = P[\bigcap_{j=1}^{i}\{S_n^{(j)}<j\}] \tag{9}$$

we have

Theorem 2. Under the conditions stated before, assume that there exists sequences of real constants $\{a_n\}_{n\ge1}$ $(a_n>0)$, $\{b_n\}_{n\ge1}$ and a non-degenerate continuous d.f. H(z) such that

$$\lim_{n\to\infty} P[M_n^{(1)}\le a_n z+b_n] = H(z),\ z\in\mathbb{R} \tag{10}$$

being additionally valid the uniformity condition for the maximum (6), and for sequences $u_n=a_n z+b_n$, $z\in\mathbb{R}$, $n\ge1$. Then, for any fixed integer i there exists a non-degenerate i-variate d.f. $H(z_1,\ldots,z_i)$ such that

$$\lim_{n\to\infty} P[\bigcap_{j=1}^{i}\{M_n^{(j)}\le a_n z_j+b_n\}] = H(z_1,\ldots,z_i) \tag{11}$$

to which corresponds, whenever $h(z)=H'(z)$ exists, a p.d.f. given by (1), g(.) and G(.) replaced by h(.) and H(.) respectively.

Notice that under the context stated before, also this theorem may be derived from theorem 3 of Weissman (1975) (taking there q=1, $t_q=1$).

In the set-up considered, with no further restrictions, H(z) is in Mejzler's class $\mathbf{M}_1$ of d.f.'s such that (cf. Galambos (1978), p. 181) either

(i) $-\log\{H(x)\}$ convex

or

(ii) $R=\sup\{x: H(x)<1\}$ finite and $-\log\{H(R-\exp(-x))\}$ convex.

Notice however that the results in theorem 2 remain valid if we work with more restrictive sequences of r.v.'s satisfying the uniformity condition for the maximum, like set-ups that lead us to the refinements $\mathbf{M}_r$, r>1, of Mejzler's class $\mathbf{M}_1 \supset \mathbf{M}_2 \supset \ldots$ introduced by Graça Martins and Pestana (1985), or to $\mathbf{M}_\infty = \bigcap_{r\geq 1} \mathbf{M}_r$. Classes $\mathbf{M}_r$, r>1, and $\mathbf{M}_\infty$ are characterized like $\mathbf{M}_1$ by (i) and (ii), with convexity replaced by monotonicity of order r, r>1, and complete monotonicity respectively.

Notice also that H_θ defined in (3) is a member of $\mathbf{M}_1$ for every $\theta \in \mathbb{R}$. More than that: H_θ belongs to $\mathbf{M}_\infty$ if and only if $\theta \geq 0$.

3. DISTRIBUTIONAL BEHAVIOUR OF TEST STATISTICS, UNDER A MULTIVARIATE EXTREMAL H_0 MODEL

We shall consider here the multivariate extremal H_θ model $\underset{\sim}{X}=(X_1,\ldots,X_{m+1})$ where $\underset{\sim}{Z} = (Z_j=(X_j-\lambda)/\delta,\ 1\leq j\leq m+1)$, $\lambda \in \mathbb{R}$, $\delta \in \mathbb{R}^+$, has a p.d.f. $h_\theta(z_1,\ldots,z_{m+1})$ given by (1), i=m+1, and G(.) replaced by the Pareto d.f. $H_\theta(.)$ in (3). As mentioned before, our interest lies in testing H_0: $\theta=0$ versus suitable one-sided or two-sided alternatives, and we first use Gumbel statistic G_{m+1} defined by (4).

Notice that G_{m+1} is invariant under location and scale transformations, i. e., $G_{m+1} = G_{m+1}(\underset{\sim}{X}) = G_{m+1}(\underset{\sim}{Z})$. Under H_0:$\theta=0$, Z_j-Z_{j+1}, $1\leq j\leq m$, are independent exponential r.v.'s, and consequently G_{m+1} is, for m even, the quotient of two Gamma(m/2) independent r.v.'s, and for m odd, the quotient of independent Gamma((m+1)/2) and Gamma((m-1)/2) r.v.'s. Consequently G_{m+1} is distributed as F(m,m), when m is even, and as (m+1)F(m+1,m-1)/(m-1) when m is odd. $F(\nu_1,\nu_2)$ denotes, as usual, the F-distribution with parameters (ν_1,ν_2). We consequently consider the test statistic

$$G^*_{m+1} = \sqrt{m}\,\{G_{m+1}-1\}/2 \tag{12}$$

which is asymptotically, as $m\to\infty$, a standard normal r.v..

For small m, tables of the F-distribution may thus be used to obtain percentage points of $G^*_{m+1} | H_0$:$\theta=0$ in the multivariate extremal H_0 model, both for one-sided and two-sided alternatives.

In the same context of statistical choice in a multivariate extremal H_θ model, an analogue of the LMP test statistic is considered. Indeed, in the standard model $\underset{\sim}{Z}=(Z_1,\ldots,Z_{m+1})$, the LMP test statistic for H_0:$\theta=0$ is, asymptotically,

$$L_{m+1}(\underset{\sim}{Z}) = \left.\frac{\partial \log h_\theta(Z_1,\ldots,Z_{m+1})}{\partial\theta}\right|_{\theta=0} = -\sum_{j=1}^{m+1} Z_j - Z^2_{m+1}/2 \tag{13}$$

for both one-sided or two-sided alternatives.

When working with the general model $\underset{\sim}{X}=(X_1>X_2>\ldots>X_{m+1})$, we consider, as usual, the test statistic

$$\hat{L}_{m+1}(\underset{\sim}{X}) = L_{m+1}((\underset{\sim}{X}-\hat{\lambda}_0\ \underset{\sim}{1})/\hat{\delta}_0) \tag{14}$$

where $(\hat{\lambda}_0,\hat{\delta}_0)$ are the maximum likelihood estimators of the unknown parameters (λ,δ) under $H_o{:}\theta=0$, $\underset{\sim}{1}$ being a column vector with all its components equal to one. Since we have

$$\hat{\lambda}_0 = X_1\ ;\ \hat{\delta}_0 = (X_1-X_{m+1})/(m+1) \tag{15}$$

we finally obtain, after a few manipulations, $\tilde{L}_{m+1}$, given by (5), an equivalent analogue of the LMP test statistic.

Since, under $H_o{:}\theta=0$, $\{Z_j-Z_{m+1}\}/\{Z_1-Z_{m+1}\}=\{\sum_{k=j}^{m} V_k\}/\{\sum_{k=1}^{m} V_k\}$, $2\leq j\leq m$, $\{V_k\}_{1\leq k\leq m}$ i.i.d. exponential r.v.'s, are the descending order statistics associated to a sample U_k, $1\leq k\leq m-1$, of i.i.d. Uniform(0,1) r.v.'s, we have the distributional identity

$$\tilde{L}_{m+1} = (m-1) - \sum_{k=1}^{m-1} U_k \tag{16}$$

The test statistic considered here is thus

$$L^*_{m+1} = \sqrt{12(m-1)}\ \{\frac{1}{m-1}\tilde{L}_{m+1} - 1/2\ \} \tag{17}$$

which is asymptotically, as $m\to\infty$, and under $H_o{:}\theta=0$, a standard normal r.v..

Since the sum of uniform r.v.'s in (16) converges fast to the normal distribution, the standard normal percentiles may be used to a very good accuracy, when dealing with the test statistic (17), even for quite small m.

4. COMPARISON OF TEST STATISTICS

Simulation of the multivariate extremal H_θ model, H_θ given by (3), is straightforward: from a set $\{R_i\}_{i\geq1}$ of pseudo random numbers in (0,1), we compute, for $1\leq j\leq m+1$

$$Z_j = H_\theta^{-1}(\prod_{k=1}^{j} R_k) = \begin{cases} \{(\prod_{k=1}^{j} R_k)^\theta-1\}/\ \theta & \text{if } \theta\neq0 \\ \\ \sum_{k=1}^{j} \log(R_k) & \text{if } \theta=0 \end{cases}$$

Comparison of test statistics is thus made by simulation. In table I we present results regarding the power functions of the statistical choice tests based on G_{m+1} and $\tilde{L}_{m+1}$ for m=20 and m=60 and for testing $H_o: \theta=0$ versus $H_1: \theta\neq 0$ in the multivariate extremal H_θ model. For each value of θ and for each test statistic we give the simulated power of that test statistic. The number of runs in each simulation was chosen such that the standard errors associated to powers are smaller than .005. Blank entries correspond to simulated powers higher than .995.

Figure 1 is a visual representation of table I, m=20.

Notice that, contrary to what happened in a multivariate GEV(θ) model, the power function of the LMP test statistic turns out to be, uniformly over $\theta \in \mathbb{R}$, higher than the power function of the Gumbel statistic, for testing $H_o:\theta=0$ in a multivariate extremal H_θ model. It is natural that, asymptotically, the same happens, since $\tilde{L}_{m+1}$ was built according to an 'optimal' asymptotic criterion whereas G_{m+1} was merely based on heuristic reasons. Asymptotic power of these statistics, for testing $H_o:\theta=0$ in a multivariate extremal H_θ model, is under investigation. It is however worth mentioning that the naïve and simple statistic G_{m+1} is practically (almost) as good as the LMP test statistic.

TABLE I. Comparative power functions of G_{m+1} and $\tilde{L}_{m+1}$ for m=20,60 and at a significance level α=.05

	m = 20		m = 60	
θ	G_{m+1}	$\tilde{L}_{m+1}$	G_{m+1}	$\tilde{L}_{m+1}$
-.30	.99			
-.25	.98	.99		
-.20	.94	.97		
-.15	.84	.89		
-.125	.73	.80		
-.1	.57	.63		
-.075	.38	.44		
-.05	.21	.24		
-.025	.09	.10	.80	.88
.025	.09	.10	.81	.89
.05	.20	.24		
.075	.38	.45		
.1	.56	.63		
.125	.72	.79		
.15	.83	.88		
.20	.93	.96		
.25	.98	.99		
.30	.99			

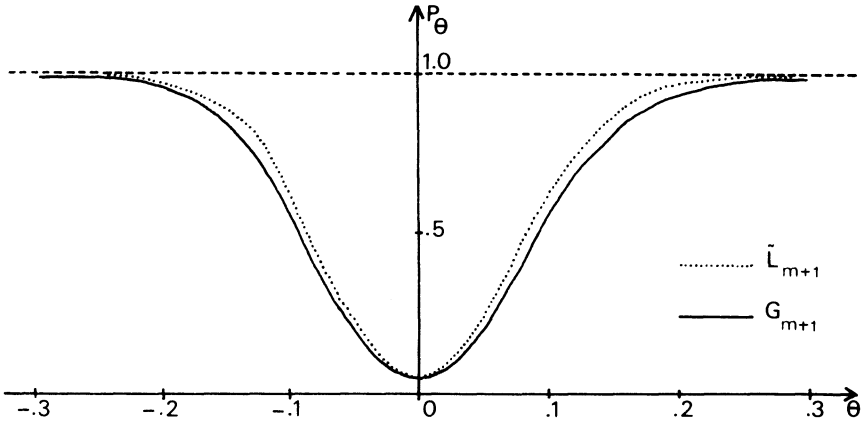

Figure 1. Power functions of G_{m+1} and $\tilde{L}_{m+1}$, m=20, α=.05

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