

# A BRIEF HISTORY AND REVIEW OF ACCELERATORS

*P.J. Bryant*

CERN, Geneva, Switzerland

## ABSTRACT

The history of accelerators is traced from three separate roots, through a rapid development to the present day. The well-known Livingston chart is used to illustrate how spectacular this development has been with, on average, an increase of one and a half orders of magnitude in energy per decade, since the early thirties. Several present-day accelerators are reviewed along with plans and hopes for the future.

## 1. INTRODUCTION

High-energy physics research has always been the driving force behind the development of particle accelerators. They started life in physics research laboratories in glass envelopes sealed with varnish and putty with shining electrodes and frequent discharges, but they have long since outgrown this environment to become large-scale facilities offering services to large communities. Although the particle physics community is still the main group, they have been joined by others of whom the synchrotron light users are the largest and fastest growing. There is also an increasing interest in radiation therapy in the medical world and industry has been a long-time user of ion implantation and many other applications. Consequently accelerators now constitute a field of activity in their own right with professional physicists and engineers dedicated to their study, construction and operation.

This paper will describe the early history of accelerators, review the important milestones in their development up to the present day and take a preview of future plans and hopes.

## 2. HISTORICAL ROOTS

The early history of accelerators can be traced from three separate roots. Each root is based on an idea for a different acceleration mechanism and all three originated in the twenties.

### 2.1 The main "History Line"

The first root to be described is generally taken as the principal "history line", since it was the logical consequence of the vigorous physics research programme in progress at the turn of the century. Indeed, particle physics research has always been the driving force behind accelerator development and it is therefore very natural to also consider high-energy physics as the birth place.

The main events along this "history line" are listed in Table 1. The line is started at the end of the last century to show the natural progression through atomic physics to nuclear physics and the inevitable need for higher energy and higher intensity "atomic projectiles" than those provided by natural radioactive sources. In this context, the particle accelerator was a planned development and it fulfilled its goal of performing the first man-controlled splitting of the atom. It was Ernest Rutherford, in the early twenties, who realised this need, but the electrostatic machines, then available, were far from reaching the necessary voltage and for a few years there was no advance. Suddenly, the situation changed in 1928, when Gurney and Gamov independently predicted tunnelling [1] and it appeared that an energy of 500 keV might just suffice to split the atom. This seemed technologically feasible to Rutherford and he immediately encouraged Cockcroft and Walton to start designing a 500 kV particle accelerator.

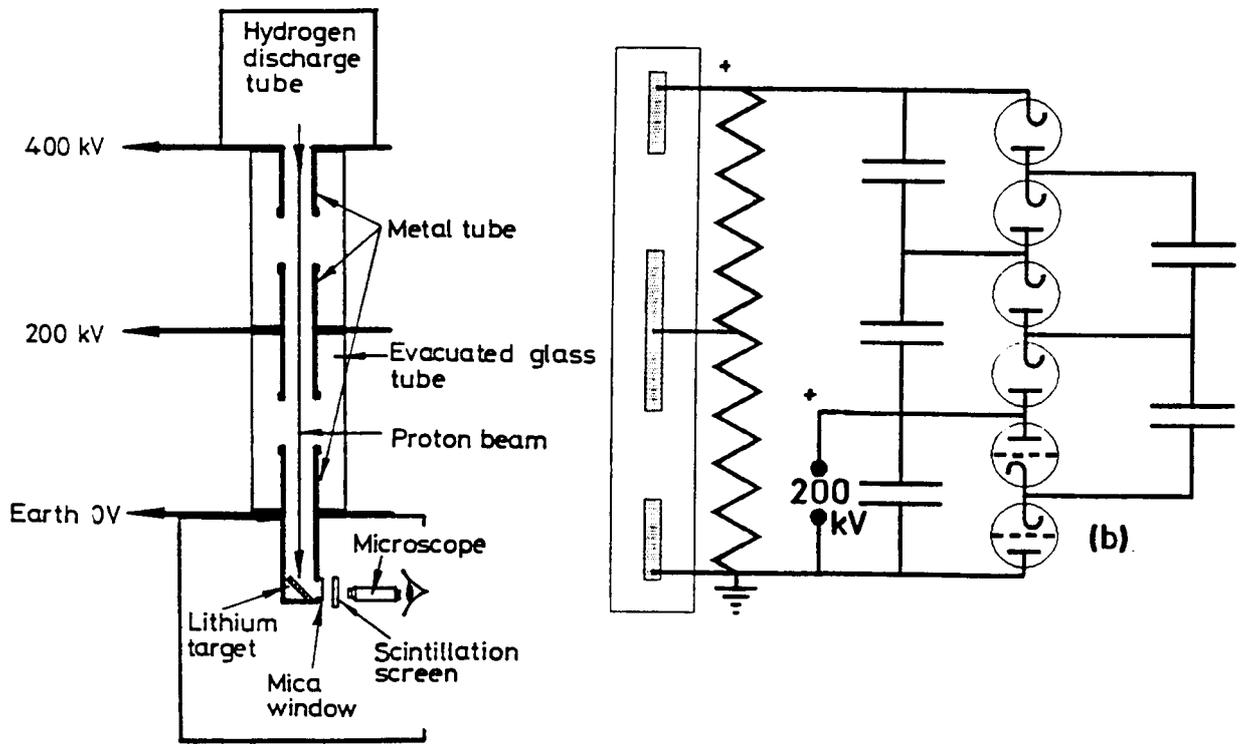
Four years later in 1932, they split the lithium atom with 400 keV protons. This was the first fully man-controlled splitting of the atom [2] which earned them the Nobel prize in 1951.

**Table 1**  
Main "History Line"

|      |   |  |
|------|---|--|
| 1895 | Lenard. Electron scattering on gases (Nobel Prize).   | < 100 keV electrons.<br>Wimshurst-type machines. |
| 1913 | Franck and Hertz excited electron shells by electron bombardment.   |  |
| 1906 | Rutherford bombards mica sheet with natural alphas and develops the theory of atomic scattering.  | Natural alpha particles of several MeV           |
| 1911 | Rutherford publishes theory of atomic structure.  |  |
| 1919 | Rutherford induces a nuclear reaction with natural alphas.  |  |
|      | ... Rutherford believes he needs a source of many MeV to continue research on the nucleus. This is far beyond the electrostatic machines then existing, but ... |  |
| 1928 | <b>Gamov predicts tunnelling and</b> perhaps 500 keV would suffice ...  |  |
| 1928 | Cockcroft & Walton start designing an 800 kV generator encouraged by Rutherford.  |  |
| 1932 | Generator reaches 700 kV and Cockcroft & Walton split lithium atom with only 400 keV protons. They received the Nobel Prize in 1951.                            |  |

Figure 1(a) shows the original apparatus, which is now kept in the Science Museum, London. The top electrode contains the proton source and was held at 400 kV, the intermediate drift tube at 200 kV and final drift tube and target at earth potential. This structure can be seen inside the evacuated glass tube in Fig. 1 above the curtained booth in which the experimenter sat while watching the evidence of nuclear disintegrations on a scintillation screen. The voltage generator, Fig. 1(b), was at the limit of the in-house technology available to Cockcroft and Walton and the design voltage of 800 kV was never reached due to a persistent spark discharge which occurred at just over 700 kV. However, the famous atom-splitting experiment was carried out at 400 kV, well within the capabilities of the apparatus. The **Cockcroft Walton generator**, as it became known, was widely used for many years after as the input stage (up to 800 kV) for larger accelerators, since it could deliver a high current.

At about the same time Van de Graaff, an American who was in Oxford as a Rhodes scholar, invented an electrostatic generator for nuclear physics research and later in Princeton, he built his first machine, which reached a potential of 1.5 MV [3]. It took some time to develop the acceleration tube and this type of machine was not used for physics research until well after the atom had been split in 1932. The principle of this type of generator is shown in Fig. 2.

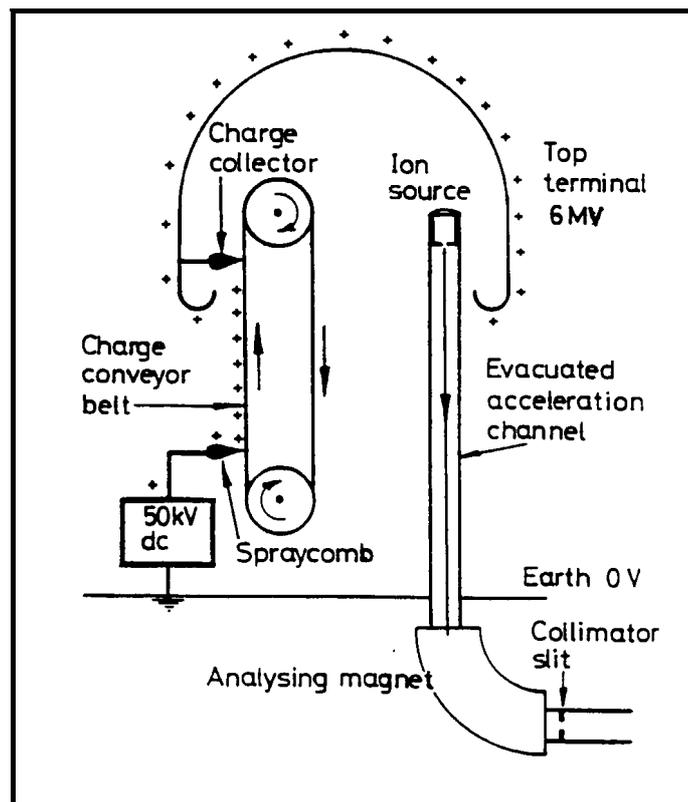


(a) Accelerating column

(b) DC generator

Fig. 1 Cockcroft and Walton's apparatus for splitting the lithium nucleus

Fig. 2 Van de Graaff electrostatic generator



Two new features appeared in later versions of the **Van de Graaff generator**. Firstly, the sparking threshold was raised by putting the electrode system and accelerating tube in a high-pressure tank containing dry nitrogen, or Freon, at 9-10 atmospheres, which enables operation typically up to 10 MV. The second was a later development, which has the special name of the **Tandem** accelerator (see Fig. 3).

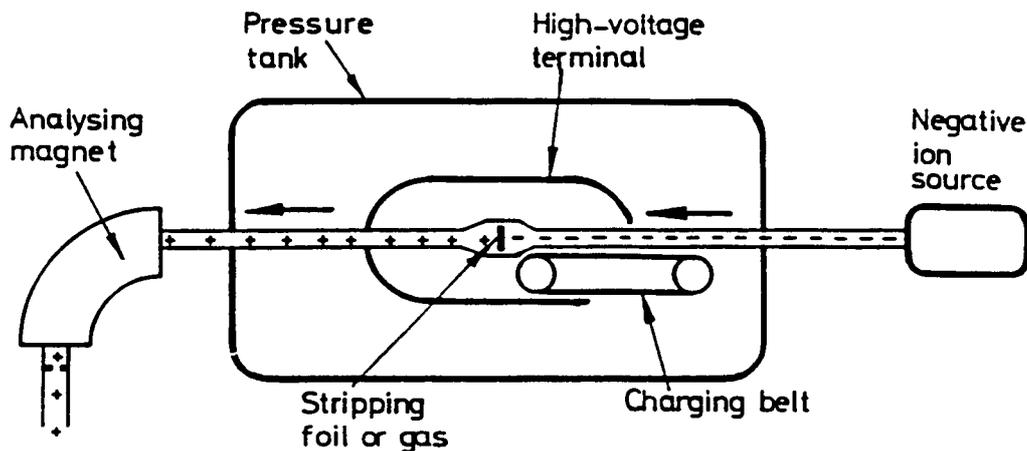


Fig. 3 Two-stage Tandem accelerator

The new feature in the Tandem accelerator was to use the accelerating voltage twice over. First an extra electron is attached to the neutral atoms to create negative ions. In recent years, a great deal of development has been done and it is now possible to obtain negative ion sources for almost all elements. The negative ion beam is injected at ground potential into the Tandem and accelerated up to the high-voltage terminal where it passes through a thin foil which strips at least two electrons from each negative ion converting them to positive ions. They are then accelerated a second time back to earth potential. The Van de Graaff generator and the Tandem provide beams of stable energy and small energy spread, but they are unable to provide as high currents as the Cockcroft-Walton generator

The highest energy Tandem is at Oak Ridge National Laboratory and routinely operates with 24.5 MV on the central terminal. However, development is not at a standstill and there is a project (the Vivitron) underway at Strasbourg to build a Tandem operating at 35 MV.

## 2.2 The second "History Line"

The direct-voltage accelerators were the first to be exploited for nuclear physics research, but they were limited to the maximum voltage that could be generated in the system (except for the astute double use of the applied voltage in the Tandem). This limitation was too restrictive for the requirements of high-energy physics and an alternative was needed.

In fact, an alternative had already been proposed in 1924 in Sweden by Ising [4]. He planned to repeatedly apply the same voltage to the particle using alternating fields and his invention was to become the underlying principle of all of today's ultra-high-energy accelerators. This is known as **resonant acceleration**. The main events along this "history line", starting with Ising, are given in Table 2.

The difference between the acceleration mechanisms of Cockcroft and Walton and Ising depend upon whether the fields are static (i.e. conservative) or time-varying (i.e. non-conservative). The electric field can be expressed in a very general form as the sum of two terms, the first being derived from a scalar potential and the second from a vector potential,

$$\vec{E} = -\nabla\phi - \frac{\partial}{\partial t}\vec{A} \quad (1)$$

where

$$\vec{B} = \nabla \times \vec{A} . \quad (2)$$

**Table 2**  
The second "History Line"

|      |   |
|------|---|
| 1924 | Ising proposes time-varying fields across drift tubes. This is "resonant acceleration", which can achieve energies above that given by the highest voltage in the system. |
| 1928 | Wideröe demonstrates Ising's principle with a 1 MHz, 25 kV oscillator to make 50 keV potassium ions.  |
| 1929 | Lawrence, inspired by Wideröe and Ising, conceives the cyclotron.   |
| 1931 | Livingston demonstrates the cyclotron by accelerating hydrogen ions to 80 keV.  |
| 1932 | Lawrence's cyclotron produces 1.25 MeV protons and he also splits the atom just a few weeks after Cockcroft and Walton (Lawrence received the Nobel Prize in 1939).       |

The first term in (1) describes the static electric field of the Cockcroft-Walton and Van de Graaff machines. When a particle travels from one point to another in an electrostatic field, it gains energy according to the potential difference, but if it returns to the original point, for example, by making a full turn in a circular accelerator, it must return to its original potential and will lose exactly the energy it has gained. Thus a gap with a DC voltage has no net accelerating effect in a circular machine.

The second term in (1) describes the time-varying field. This is the term that makes all the present-day high-energy accelerators function. The combination of (1) and (2) yields Faraday's law,

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B} ,$$

which relates the electric field to the rate of change of the magnetic field. There are two basic geometries used to exploit Faraday's Law for acceleration. The first of which is the basis of Ising's idea and the second "history line", and the second is the basis of the third "history line" to be described later.

Ising suggested accelerating particles with a linear series of conducting drift tubes and Wideröe built a 'proof-of-principle' linear accelerator in 1928 [5]. Alternate drift tubes are connected to the same terminal of an RF generator. The generator frequency is adjusted so that a particle traversing a gap sees an electric field in the direction of its motion and while the particle is inside the drift tube the field reverses so that it is again the direction of motion at the next gap. As the particle gains energy and speed the structure periods must be made longer to maintain synchronism (see Fig. 4).

Clearly, as the velocity increases the drift tubes become inconveniently long, unless the frequency can be increased, but at high frequencies the open drift-tube structure is lossy. This problem is overcome by enclosing the structure to form a cavity (in a circular machine) or series

of cavities (in a linear machine), working typically in the MHz range. The underlying principle remains unchanged, but there are several variants of the accelerating structure design.

Ising's original idea can be considered as the beginning of the 'true' **accelerator**. Indeed, the next generation of linear colliders, which will be in the TeV range, will probably still be applying his principle of resonant acceleration, except that the frequency will probably be in the tens of GHz range.

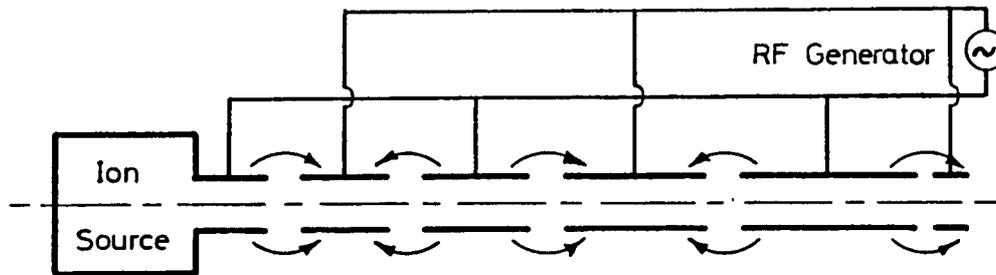


Fig. 4 RF linac

Technologically the linear accelerator, or **linac** as it is known, was rather difficult to build and, during the 1930's, it was pushed into the background by a simpler idea conceived by Ernest Lawrence in 1929 [6], the fixed-frequency cyclotron (see Fig. 5). Lawrence's idea was inspired by a written account of Wideröe's work and M. Livingston demonstrated the principle by accelerating hydrogen ions to 80 keV in 1931. Lawrence's first model worked in 1932 [7]. It was less than a foot in diameter and could accelerate protons to 1.25 MeV. He split the atom only weeks after Cockroft and Walton. Lawrence received the Nobel Prize in 1939, and by that year the University of California had a 5-foot diameter cyclotron (the 'Crocker' cyclotron) capable of delivering 20 MeV protons, twice the energy of the most energetic alpha particles emitted from radioactive sources. The cyclotron, however, was limited in energy by relativistic effects and despite the development of the synchrocyclotron, a new idea was still required to reach yet higher energies in order to satisfy the curiosity of the particle physicists. This new idea was to be the **synchrotron**, which will be described later.

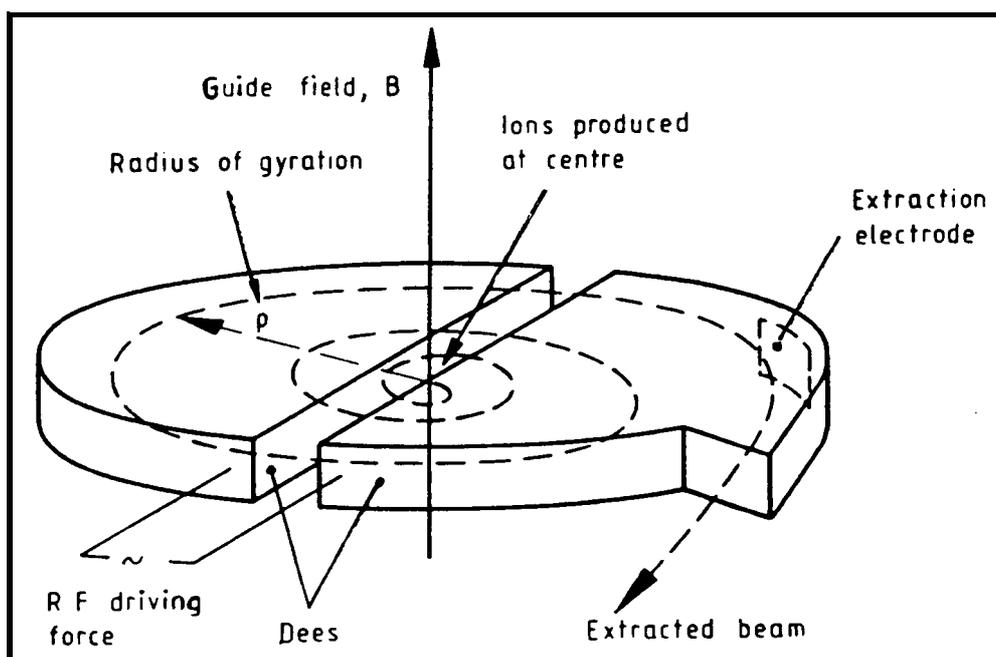


Fig. 5 Schematic cyclotron

### 2.3 The third and fainter 'History Line'

In the previous section, it was mentioned that there were **two** equipment configurations for exploiting Faraday's Law for acceleration. First, consider the application of Faraday's Law to the linac, which is made more evident by enclosing the gaps in cavities. For simplicity the fields in a single RF cavity are shown schematically in Fig. 6(a).

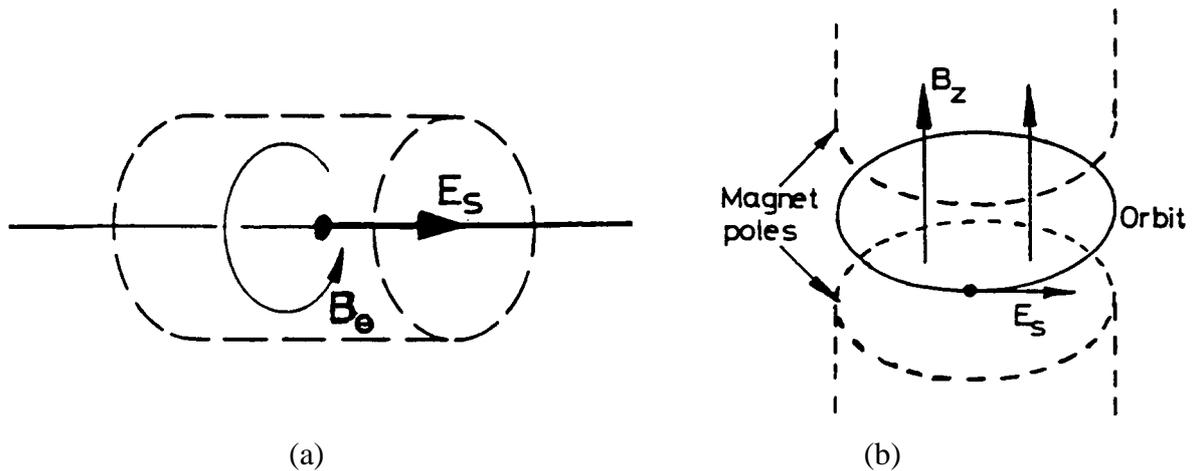


Fig. 6 Acceleration configurations

The azimuthal magnetic field is concentrated towards the outer wall and links the beam. Faraday's Law tells us the periodic rise and fall of this magnetic field induces an electric field on the cavity axis, which can be synchronised with the passage of the beam pulse.

Suppose now that the topology is transformed, so that the beam encircles the magnetic field as shown in Fig. 6(b). Wideröe [8, 9] suggested this configuration and the acceleration mechanism, now known as "betatron acceleration". He called his idea a "strahlung transformer" or "ray transformer", because the beam effectively formed the secondary winding of a transformer (see Figs. 6 and 7)). As the flux through the magnet core is increased, it induces an azimuthal e.m.f. which drives the charged beam particles to higher and higher energies. The trick is to arrange for the increase in the magnetic field in the vicinity of the beam to correspond to the increase in particle energy, so that the beam stays on the same orbit\*. This device, the betatron, is insensitive to relativistic effects and was therefore ideal for accelerating electrons. The betatron has also the great advantages of being robust and simple. The one active element is the power converter that drives the large inductive load of the main magnet. The focusing and synchronisation of the beam energy with the field level are both determined by the geometry of the main magnet. As noted in the third "history line" in Table 3, Wideröe put this idea in his laboratory notebook, while he was a student, but it remained unpublished only to re-surface many years later when Kerst [10] built the first machine of this type. When in 1941 Kerst and Serber published a paper on the particle oscillation in their betatron [11], the term "betatron oscillation" became universally adopted for referring to such oscillations in all devices.

\* Known as the Wideröe condition, or 2-to-1 rule.

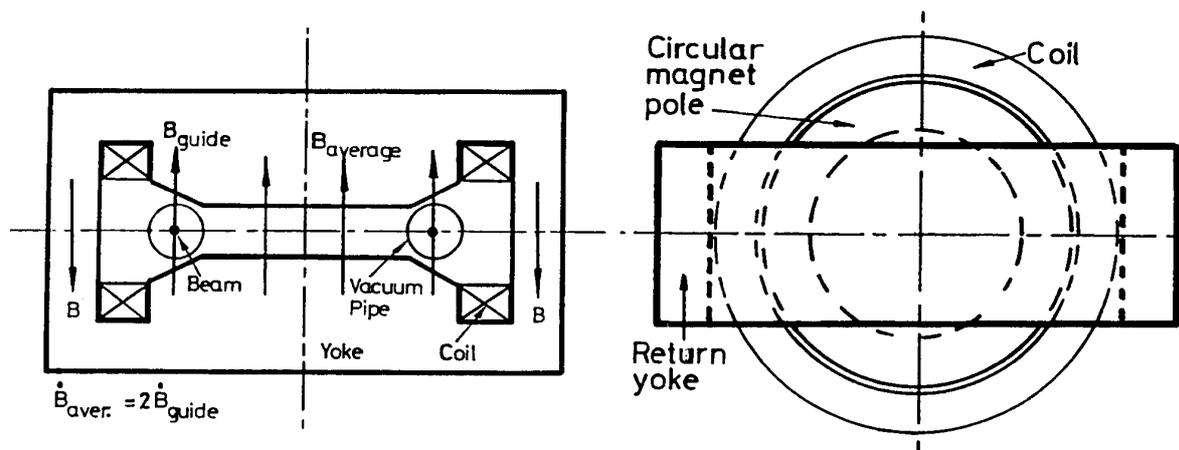


Fig. 7 Strahlung transformator or betatron

**Table 3**  
The third "History Line"

|      |   |
|------|---|
| 1923 | Wideröe, a young Norwegian student, draws in his laboratory notebook the design of the betatron with the well-known 2-to-1 rule. Two years later he adds the condition for radial stability <b>but does not publish</b> . |
| 1927 | Later in Aachen Wideröe makes a model betatron, but it does not work. Discouraged he changes course and builds the linear accelerator mentioned in Table 2.   |
| 1940 | Kerst re-invents the betatron and builds the first working machine for 2.2 MeV electrons.   |
| 1950 | Kerst builds the world's largest betatron of 300 MeV.   |

The development of betatrons for high-energy physics was short, ending in 1950 when Kerst built the world's largest betatron (300 MeV), but they continued to be built commercially for hospitals and small laboratories where they were considered as reliable and cheap. In fact the betatron acceleration mechanism is still of prime importance. In the present-day synchrotron, there is a small contribution to the beam's acceleration which arises from the increasing field in the main dipoles. If an accurate description of the longitudinal motion is required, then the betatron effect has to be included.

### 3. THE MAIN DEVELOPMENT

By the 1940's three acceleration mechanisms had been demonstrated:- **DC acceleration, resonant acceleration and the betatron mechanism**. In fact, there were to be no new ideas for acceleration mechanisms until the mid-1960's, when collective acceleration [12] was proposed in which heavy ions are accelerated in the potential well of an electron ring and the 1980's when there were several Workshops devoted entirely to finding new acceleration techniques. However, the acceleration mechanism is not sufficient by itself and other equally important developments are needed.

In order to accelerate particles to very high energies, it is also necessary to have focusing mechanisms in the transverse and longitudinal (energy) planes. This was not always appreciated. In the early cyclotrons, for example, the field was made as uniform as possible only to find that the beam was unstable. Livingston [13] who was Lawrence's research student, told how they shimmed the magnet for each small step in energy to keep the beam stable, thus ending up with a field shape for transverse stability that decreased with radius. Theory has later shown that this decrease should be an inverse power law of the radius between zero and unity.

The cyclotron is limited by relativistic effects, which cause the particles to slow down and lose synchronism with the RF field. At first glance it would appear that one would only have to reduce the frequency in order to maintain synchronism, but this is a little too naïve since the spread in revolution frequency with energy would quickly exploit the natural energy spread in the beam and disperse the particles away from the peak of the RF voltage. In this case a longitudinal focusing mechanism is needed. This problem was overcome by E. McMillan [14] and independently by V. Veksler [15] who discovered the principle of **phase stability** in 1944 and invented the **synchrotron**. Phase stability is general to all RF accelerators except the fixed-frequency cyclotron. The effect is that a bunch of particles, with an energy spread, can be kept bunched throughout the acceleration cycle by simply injecting them at a suitable phase of the RF cycle. This focusing effect was strong enough that the frequency modulation in the synchro-cyclotron did not have to be specially tailored and was simply sinusoidal. Synchro-cyclotrons can accelerate protons to about 1 GeV, a great improvement on the simple cyclotron, but the repetition rate reduces the particle yield.

In the synchrotron [14, 15] the guide field increases with particle energy, so as to keep the orbit stationary as in the betatron, but acceleration is applied with an RF voltage via a gap or cavity. In 1946 F. Goward and D. Barnes [16] were the first to make a synchrotron work, and in 1947 M. Oliphant, J. Gooden and G. Hyde [17] proposed the first proton synchrotron for 1 GeV in Birmingham, UK. However, the Brookhaven National Laboratory, USA, built their 3 GeV Cosmotron by 1952, just one year ahead of the Birmingham group.

Up to this time the only mechanism known for focusing in the transverse plane was called **weak, or constant-gradient focusing**. In this case, the guide field decreases slightly with increasing radius and its gradient is constant all round the circumference of the machine. The tolerance on the gradient is severe and sets a limit to the size of such an accelerator. The aperture needed to contain the beam also becomes very large and the magnet correspondingly bulky and costly. In the early fifties the limit was believed to be around 10 GeV.

In the same year as the Cosmotron was finished (1952) E. Courant, M. Livingston and H. Snyder [18] proposed **strong focusing**, also known as **alternating-gradient (AG) focusing**. The idea had been suggested earlier by Christofilos [19] but it was not published. This new principle revolutionized synchrotron design, allowing smaller magnets to be used and higher energies to be envisaged. It is directly analogous to a well-known result in geometrical optics, that the combined focal length  $F$  of a pair of lenses of focal lengths  $f_1$  and  $f_2$  separated by a distance  $d$  is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}.$$

If the lenses have equal and opposite focal lengths,  $f_1 = -f_2$  and the overall focal length  $F = f^2/d$ , which is always positive. In fact,  $F$  remains positive over quite a large range of values when  $f_1$  and  $f_2$  have unequal values but are still of opposite sign. Thus within certain limits a series of alternating lenses will focus. Intuitively one sees that, although the beam may be defocused by one lens, it arrives at the following lens further from the axis and is therefore focused more strongly. Structures based on this principle are referred to as AG structures.

The synchrotron quickly overshadowed the synchrocyclotron and the betatron in the race for higher energies. The adoption of alternating gradient focusing for machines and transfer lines was even quicker. CERN for example immediately abandoned its already-approved project for a 10 GeV/c weak focusing synchrotron in favour of a 25 GeV/c AG machine, which it estimated could be built for the same price.

The next step was the **storage ring collider**. In physics experiments, the useful energy for new particle production is the energy that is liberated in the centre-of-mass system. When an accelerator beam is used on a fixed target, only a fraction of the particle's energy appears in the centre-of-mass system, whereas for two equal particles in a head-on collision, all of the particles' energy is available. This fundamental drawback of the fixed-target accelerator becomes more punitive as the energy increases. For example, it would have needed a fixed-target accelerator of over 1TeV to match the centre-of-mass energy available in the CERN ISR (2 x 26 GeV proton colliding rings).

The storage-ring collider now dominates the high-energy physics field. Single-ring colliders, using particles and antiparticles in the same magnetic channel, were the first type of collider to be exploited at Frascati in the AdA (Anelli di Accumulazione) project (1961). The first double-ring proton collider was the CERN ISR (Intersecting Storage Rings), 1972-1983. The highest-energy collisions obtained to date are 2 x 900 GeV in the Fermilab, single-ring, proton-antiproton collider.

Colliders have been very successful as physics research instruments. The  $J/\psi$  particle was discovered at SPEAR by B. Richter and at the same time by Ting at BNL – they shared the 1976 Nobel Prize. The CERN proton-antiproton storage ring was also the source of a Nobel Prize for C. Rubbia and S. van der Meer in 1984, following the discovery of the W and Z particles. The proton-antiproton colliders were only made possible by the invention of **stochastic cooling** by S. van der Meer for the accumulation of the antiprotons [20].

The use of **superconductivity** in proton machines has made the very highest energies possible. There has also been another change taking place, which has been called the **Exo-geographical transition** (a phrase coined by Professor N. Cabibbo at a Workshop held at Frascati in 1984). This refers to the arrangements that have made it possible to bury the very large machines such as LEP and HERA deep under property which does not belong to the laboratory concerned. Without such agreements, Europe could not have maintained its leading position in the world accelerator league.

In order to fill in some of the bigger gaps in this brief history, it is now necessary to jump back in time to mention some of the other accelerators, which may not have featured as a high-energy machine, but have found their place as injectors or as being suitable for some special application.

The **microtron**, sometimes known as the electron cyclotron, was an ingenious idea due to Veksler (1945). The electrons follow circular orbits of increasing radius, but with a common tangent. An RF cavity positioned at the point of the common tangent supplies a constant energy increment on each passage. The relativistic mass increase slows the revolution frequency of the electrons, but by a constant increment on each passage. If this increment is a multiple of the RF oscillator frequency, the electrons stay in phase, but on a different orbit. Microtrons operate at microwave frequencies and are limited to tens of MeV. They are available commercially and are sometimes used as an injector to a larger machine.

The **radio-frequency quadrupole** (RFQ) suggested in 1970 by I. Kapchinski and V. Telyakov is useful at low energies and is increasingly replacing the Cockcroft-Walton as injector. The RFQ combines focusing and acceleration in the same RF field.

The electron storage rings have given birth to the synchrotron radiation sources, more usually referred to as **light sources**. These machines are now the fastest growing community in the accelerator world and the first commercially available compact synchrotron light source for lithography has just come onto the market.

The linear accelerator was eclipsed during the thirties by circular machines. However, the advances in ultra-high frequency technology during World War II (radar) opened up new possibilities and renewed interest in linac structures. Berkeley was first, with a proton linear accelerator of 32 MeV built by Alvarez in 1946. The **Alvarez accelerator** has become very popular as an injector for large proton and heavy-ion synchrotrons all over the world with energies in the range of 50–200 MeV, that is essentially non-relativistic particles. The largest proton linear accelerator to date is the 800 MeV 'pion factory' (LAMPF) at Los Alamos.

The first electron linear accelerators were studied at Stanford and at the Massachusetts Institute for Technology (MIT) in 1946. This type of accelerator has also had a spectacular development, up to the largest now in operation, the 50 GeV linear accelerator at the Stanford Linear Accelerator Centre (SLAC). Like betatrons they have become very popular in fields outside nuclear physics, particularly for medicine.

The Livingston chart (see Fig. 8) shows, in a very striking way, how the succession of new ideas and new technologies has relentlessly pushed up accelerator beam energies over five decades at the rate of over one and a half orders of magnitude per decade. One repeatedly sees a new idea, which rapidly increases the available beam energy, but only to be surpassed by yet another new idea. Meanwhile the first idea continues into saturation and possibly into quasi-oblivion.

This brings the section on the main development almost up to date, except for the Stanford Linear Collider (SLC), but this will be mentioned under future accelerators where it fits more naturally.

#### **4. THE CURRENT SITUATION IN HIGH-ENERGY PARTICLE PHYSICS ACCELERATORS**

Table 4 contains a selection of the main operating high-energy physics machines, those under construction and those under study. The latter two groups encompass the extremes of machines like RHIC [22], which are partially constructed and the linear colliders, which are very futuristic.

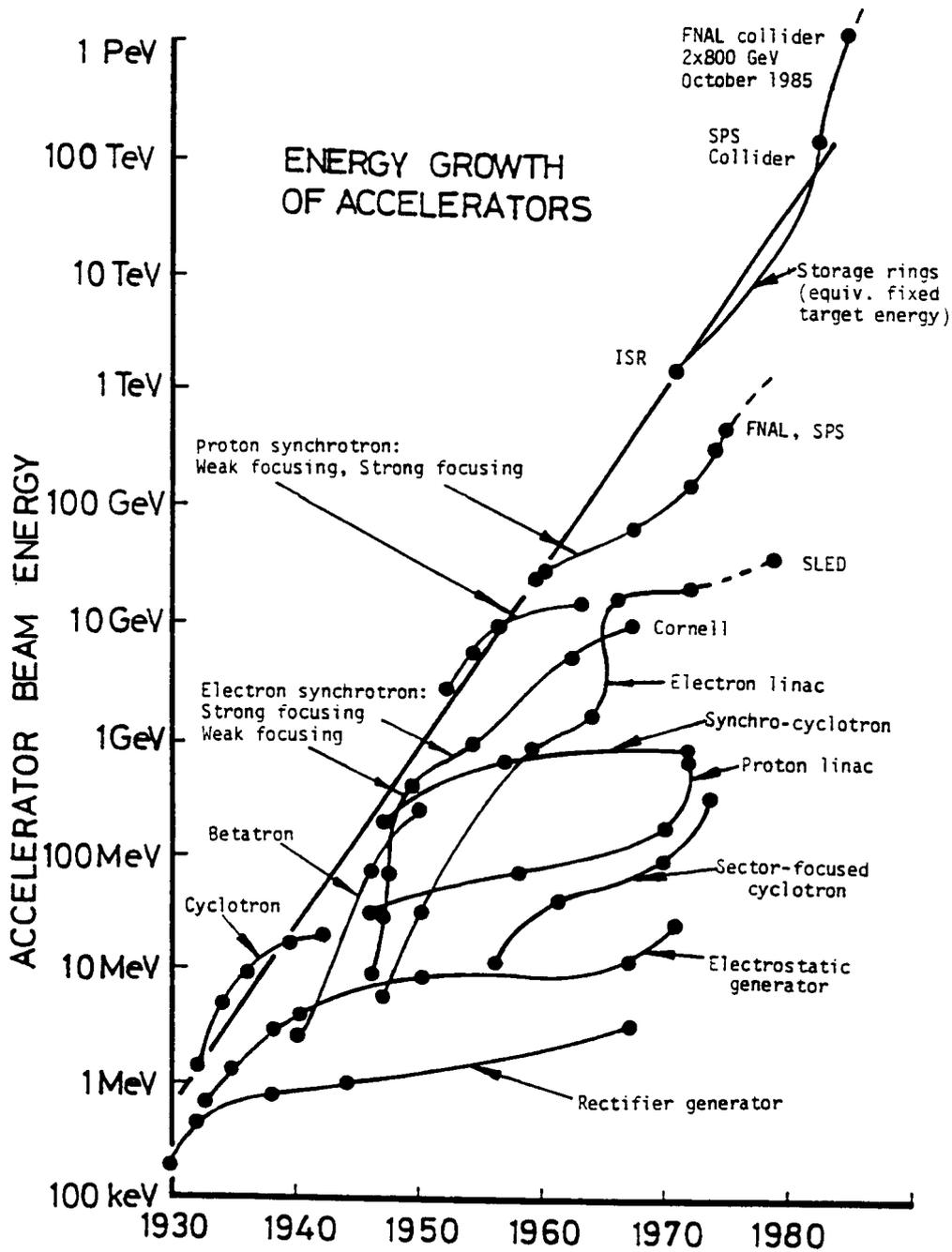


Fig. 8 Livingston chart [21]

In the present situation circular colliders dominate the high-energy field. The proton-antiproton colliders are now mature machines and it is unlikely that the USA or Western Europe will propose further facilities of this type. The technologies of stochastic and electron cooling that were developed for this class of facility are now being applied in smaller storage rings.

Once LEP [23] has been upgraded to around 100 GeV, it will almost certainly be the highest energy electron ring to be built, since the penalty to be paid in RF power to compensate the synchrotron radiation loss is already prohibitive at this energy. The solution is to change to **linear electron colliders**; a solution that was already foreseen in 1965 by Tigner [24]. The Stanford Linear Collider (SLC) [25] is a test bed for these future machines.

**Table 4**

**Operating high-energy physics accelerators**

| Accelerator         | Particles                         | Beam energy [GeV]                 | c.m. energy [GeV] | Luminosity [ $\text{cm}^{-2} \text{s}^{-1}$ ] | Remarks                         |
|---------------------|-----------------------------------|-----------------------------------|-------------------|---|---------------------------------|
| KEK<br>Japan        | p                                 | 12                                | 5                 | -   | Fixed target                    |
| AGS<br>Brookhaven   | p                                 | 33                                | 8                 | -   | Fixed target<br>Polarised p     |
| PS<br>CERN          | p<br>$e^+$ , $e^-$ , $p^-$ , ions | 28 (p)<br>3.5 (e)                 | 7<br>-            | -<br>-  | Fixed target<br>Injector        |
| CESR<br>Cornell     | $e^+$ , $e^-$                     | 9                                 | 18                | $10^{32}$                                     | Collider                        |
| Tevatron II<br>FNAL | p<br>p, $p^-$                     | 800 (p)                           | 40<br>-           | -<br>-  | Fixed target<br>Injector        |
| SPS<br>CERN         | p, e<br>p, $p^-$                  | 450 (p), 20 (e)<br>$2 \times 315$ | 30 (p),-<br>630   | -<br>$3 \times 10^{30}$                       | F. target, injector<br>Collider |
| SLC<br>SLAC         | $e^+$ , $e^-$                     |                                   | 100               | $(6 \times 10^{30})$                          | Linear<br>collider              |
| Tevatron I<br>FNAL  | p, $p^-$                          | 900                               | 1800              | $10^{31}$                                     | s.c. collider                   |
| TRISTAN<br>in Japan | $e^+$ , $e^-$                     | 32                                | 64                | $8 \times 10^{31}$                            | Collider<br>s.c. cavities       |
| LEP I<br>CERN       | $e^+$ , $e^-$                     | 55                                | 110               | $1.6 \times 10^{31}$                          | Collider                        |
| HERA<br>DESY        | e, p                              | 30 ( $e^-$ )<br>820 (p)           | 310               | $3 \times 10^{31}$                            | Collider<br>s.c. p-ring         |

**High-energy physics accelerators under construction**

|                    |               |                    |                   |  |                                  |
|--------------------|---------------|--------------------|-------------------|--|----------------------------------|
| UNK I<br>USSR      | p             | 400                | 28                | -  | Fixed target<br>Conventional     |
| SSC<br>USA         | p, p          | 20                 | 40                | $\sim 10^{33}$                             | s.c. collider                    |
| LEP II             | $e^+$ , $e^-$ | 100                | 200               | $10^{32}$                                  | Collider, s.c.<br>cavity upgrade |
| RHIC<br>Brookhaven | p to Au       | 0.25 to<br>0.1/amu | 0.5 to<br>0.2/amu | $3 \times 10^{30}$<br>$1.2 \times 10^{27}$ | s.c. collider<br>for heavy ions  |

**High-energy physics accelerators under study**

|                |                  |         |       |   |                                  |
|----------------|------------------|---------|-------|---|----------------------------------|
| UNK II<br>USSR | p, p<br>p, $p^-$ | 3       | 6     | $\sim 4 \times 10^{32}$<br>$\sim 10^{37}$ | s.c. collider<br>for 1996        |
| LHC<br>CERN    | p, p             | 8       | 16    | $\sim 10^{34}$                            | s.c. collider                    |
| CLIC<br>CERN   | $e^+$ , $e^-$    | 1       | 2     | $\sim 10^{33}$                            | Linear<br>collider               |
| SC<br>Stanford | $e^+$ , $e^-$    | 0.5 (1) | 1(2)  | $\sim 10^{33}$                            | Linear collider<br>proposal 1990 |
| VLEPP<br>USSR  | $e^+$ , $e^-$    | 0.5 (1) | 1 (2) | $\sim 10^{33}$                            | Linear collider<br>for 1996      |
| JLC<br>Japan   | $e^+$ , $e^-$    | 0.5     | 1     | $\sim 10^{33}$                            | -                                |

At the moment, the proton community is poised to build the SSC (Super Superconducting Collider) in Texas [26] and the LHC (Large Hadron Collider) in CERN [27]. Both machines are superconducting and of very large dimensions. At present there is no hard limitation on the size of hadron colliders, except of course cost. However, synchrotron radiation is already a bothersome heat load in these machines and will be a very real problem in machines of the size of the Elosatron [28] for example. The LHC is a high technology project, which will use high-field magnets (approaching 10 T) with probably 'niobium-titanium' technology at 2 K in the arcs and niobium-tin technology at 4 K in the insertions. The magnets will also be of the twin-bore design first proposed by Blewett [29].

## 5. CONCLUSION

Led by the example of the SLC, accelerator builders are now tackling formidable theoretical and technological problems in all stages of the accelerator design. In the next generation of proposed linear electron colliders the typical values required for the normalised emittance are of the order of  $10^{-7}$  rad.m. Beam sizes at the interaction point will have to be around 1 to 30 nm high with pulse lengths of 200-800  $\mu\text{m}$ . With a few  $10^0$  particles per bunch and a repetition rate of over 1000 Hz the beam power is then a few MW. The luminosity in such designs is around a few times  $10^{33}$   $\text{cm}^{-2}$   $\text{s}^{-1}$ , far higher than anything that has yet been achieved. Stability of the supporting structures and power converters driving the final focus become critically important with such small beam sizes and the fabrication of elements such as the final focus quadrupoles requires new techniques.

At present the linear collider designs are called quasi-conventional. For example, the CERN CLIC study [30] assumes the use of a warm copper accelerating structure operating at 29 GHz giving 80 MV/m. If this sounds easy, then consider that the structure will be powered from a superconducting drive linac. Such high-gradient, high-frequency structures have never before been used and neither has a superconducting linac been used in this way to drive a second accelerator. In fact, the term "quasi-conventional" is really a misnomer.

The future holds many challenges for the accelerator engineer both in the gigantic superconducting hadron machines now proposed and in the new generation of electron linear colliders.

\* \* \*

## REFERENCES

- [1] R.W. Gurney and E.U. Condon, Nature 122, 439 (1928) and G. Gamov, Zeit f. Phys. 51, 204 (1928).
- [2] J.D. Cockcroft and E.T.S. Walton, "Experiments with high velocity ions", Proc. Royal Soc., Series A 136 (1932), 619-30.
- [3] R.J. Van de Graaff, "A 1,500,000 volt electrostatic generator", Phys. Rev., 387, (Nov. 1931), 1919-20.
- [4] G. Ising, Arkiv för Matematik, Astronomi och Fysik, 18 (1924), 1-4.
- [5] R. Wideröe, Arch.für Elektrotechnik, 21 (1928), 387-406.

- [6] E.O. Lawrence and N.E. Edlefsen, *Science*, 72 (1930), 376–7.
- [7] E.O. Lawrence and M.S. Livingston, "The production of high speed light ions without the use of high voltages", *Phys. Rev.*, 40 (April 1932), 19–35.
- [8] W. Paul, "Early days in the development of accelerators", *Proc. Int. Symposium in Honour of Robert R. Wilson, Fermilab, 1979* (Sleepeck Printing Co. Bellwood, Ill., 1979), 25–688.
- [9] R. Wideröe, "Some memories and dreams from the childhood of particle accelerators", *Europhysics News*, Vol. 15, No. 2 (Feb. 1984), 9–11.
- [10] D.W. Kerst, "The acceleration of electrons by magnetic induction", *Phys. Rev.*, 60 (July 1942), 47–53.
- [11] D.W. Kerst and R. Serber, "Electronic orbits in the induction accelerator", *Phys. Rev.*, 60 (July 1941), 53–58.
- [12] G.S. James, R.H. Levy, H.A. Bethe and B.T. Fields, "On a new type of accelerator for heavy ions", *Phys. Rev.*, 145, 925 (1966).
- [13] M.S. Livingston, "Particle accelerators: A brief history", Harvard University Press, Cambridge, Massachusetts (1969), 32.
- [14] E.M. McMillan, "The synchrotron - a proposed high-energy particle accelerator", *Phys. Rev.*, Letter to the editor, 68 (Sept. 1945), 1434.
- [15] V. Veksler, *J. of Phys, USSR*, 9 (1945), 153–8.
- [16] F.K. Goward and D.E. Barnes, *Nature*, 158 (1946), 413.
- [17] M.L. Oliphant, J.S. Gooden and G.S. Hyde, *Proc. Phys. Soc.*, 59 (1947), 666.
- [18] E.D. Courant, M.S. Livingston and H.S. Snyder, "The strong-focusing synchrotron – a new high-energy accelerator", *Phys. Rev.*, 88 (Dec. 1952), 1190-6, and E.D. Courant and H.S. Snyder, "Theory of the alternating-gradient synchrotron", *Annals of Physics*, No. 3 (1958), 1–48.
- [19] N.C. Christofilos, Unpublished report (1950), and U.S. Patent no. 2.736,799, filed March 10, 1950, issued February 28, 1956.
- [20] S. van der Meer, "Stochastic damping of betatron oscillations in the ISR", CERN/ISR-PO/72–31 (August, 1972).
- [21] Livingston chart. First published in Livingston's book: "High-energy accelerators", Interscience Publishers Inc., New York (1954).
- [22] H. Hahn, "The relativistic heavy ion collider at Brookhaven", *Proc. 1st European Part. Accel. Conf. (EPAC), Rome 1988* (World Scientific, Singapore, 1989), 109–11.
- [23] E. Picasso, "The LEP project", *Proc. 1st European Part. Accel. Conf. (EPAC), Rome 1988* (World Scientific, Singapore, 1989) 3–6.
- [24] M. Tigner, "A possible apparatus for electron clashing-beam experiments", *Letter to the editor, Nuovo Cim.*, 37 (1965), 1228–31.

- [25] R. Stiening, "The status of the Stanford Linear Collider", Proc. 1987 IEEE Part. Accel. Conf., Washington, March 1987, (IEEE, 1987), 1–7.
- [26] J. Peoples, "The SSC Project", Ibid 237–41.
- [27] G. Brianti, "The large hadron collider (LHC) in the LEP tunnel", Ibid. 218–222.
- [28] K. Johnsen, "Long term future", ECFA Study Week on Instr. Tech. for High Luminosity Hadron Colliders, Barcelona, 1989, CERN 89–10, ECFA 89–124, Vol. 1 (Nov. 1989), 25-34.
- [29] J.P. Blewett, "200 TeV Intersecting Storage Accelerators", 8th Int. Conf. on High-Energy Accel., CERN, 1971 (CERN, Geneva, 1971), 501–4.
- [30] W. Schnell, "Linear Collider Studies", Proc. 1st European Part. Accel. Conf. (EPAC), Rome 1988 (World Scientific, Singapore, 1989), 285–89.

# BASIC COURSE ON ACCELERATOR OPTICS

J. Rossbach, Deutsches Elektronen-Synchrotron DESY

P. Schmüser, II. Institut für Experimentalphysik, Universität Hamburg, F.R. Germany

## Abstract

The main subject of this introductory course is transverse optics in cyclic high energy accelerators. It is based on three one-hour lectures on “Basic transverse optics” given at the CERN Accelerator Schools (P. Schmüser, 1986 [5], 1988, 1990, and J. Rossbach, 1992). Although the emphasis is on periodic solutions and their stability, application of the formalism to non-periodic structures is also treated. We have not expanded the material beyond what can be covered in 3 lectures, but the proofs are done more carefully and some examples and details have been added which might be useful in practice.

After a brief introduction to the concepts of both weak and strong focusing of relativistic particle beams, types of magnetic lenses are discussed, followed by a careful treatment of the multipole expansion of transverse magnetic fields. Linear transverse single-particle dynamics is treated both in terms of transfer matrices and betatron oscillations. Transfer matrices of the most common accelerator lattice modules are explicitly given, including a brief introduction to quadrupole doublet imaging. Dispersion is treated, but no linear coupling. The concept of beam emittance, including Liouville’s Theorem, is discussed from different points of view. Also effects of linear field errors, stability criteria and chromaticity are dealt with. FODO cells are treated in various respects. Finally, strong and weak focusing are compared quantitatively.

## 1 INTRODUCTION

In any kind of accelerator there is exactly one curve – the design orbit – on which ideally all particles should move, see Fig. 1. If this design orbit is curved, which may be required for many reasons, *bending* forces are needed.

In reality, most particles of the beam will deviate slightly from the design orbit. In order to keep these deviations small on the whole way (which might be as long as  $10^{11}$  km in a storage ring), *focusing* forces are required. Both bending and focusing forces can be accomplished with electromagnetic fields. The Lorentz force is

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

For velocities  $v \approx c$ , a moderate magnetic field of 1 Tesla corresponds to a huge electric field of  $3 \cdot 10^8$  V/m. Therefore only transverse magnetic fields are considered.

Now consider the total energy  $E$  of a particle:

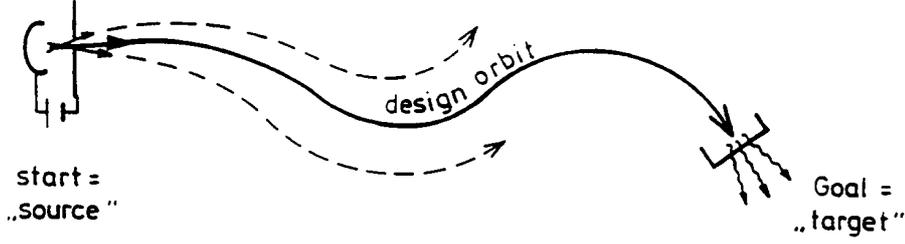


Figure 1: To guide particles on a curved design orbit, bending elements are required.

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic mechanics describes its rate of change in the presence of an external force by

$$\frac{dE}{dt} = \mathbf{v} \cdot \mathbf{F}$$

Since  $\mathbf{v}$  is perpendicular to  $\mathbf{v} \times \mathbf{B}$ , the energy and the relativistic mass  $m = m_0 / \sqrt{1 - v^2/c^2}$  are not changed in static magnetic fields. So the equation of motion is

$$\dot{\mathbf{v}} = \frac{e}{m} \mathbf{v} \times \mathbf{B} \quad (1.1)$$

The particles have to make a large number of revolutions in a circular accelerator or storage ring. Stability of motion is an important criterion and puts stringent requirements on the magnetic field in the vicinity of the equilibrium orbit. Depending on the magnitude of the focusing forces we can distinguish between weak and strong focusing.

The first machines were built with weak focusing and a guide field which does not depend on the azimuthal angle. If  $\mathbf{B}$  is independent of the azimuth angle  $\Theta$  the equilibrium orbit is a circle of radius  $\rho = mv/eB$  which we choose to be in the plane  $z = 0$ . The motion is stable if for small deviations of the particles from this orbit restoring forces arise which lead to oscillations around the orbit. These are called betatron oscillations.

Consider the motion in the horizontal plane  $z = 0$  containing the equilibrium orbit (Fig. 2). Stability requires that the Lorentz force is smaller than the centrifugal force for  $r < \rho$  and larger than it for  $r > \rho$

$$evB_z(r) \begin{cases} < \frac{mv^2}{r} & \text{for } r < \rho \\ > \frac{mv^2}{r} & \text{for } r > \rho \end{cases} \quad (1.2)$$

For small deviations from the equilibrium orbit we have

$$r = \rho + x = \rho \left( 1 + \frac{x}{\rho} \right) \quad (1.3)$$

$$\frac{mv^2}{r} \approx \frac{mv^2}{\rho} \left( 1 - \frac{x}{\rho} \right)$$

$$evB_z(r) \approx ecB_0 \left( 1 - n \frac{x}{\rho} \right) \quad (1.4)$$

with  $B_0 = B_z(\rho)$ . In Equation (1.4) we have introduced the *field index*  $n$

$$n = -\frac{\rho}{B_0} \left( \frac{\partial B_z}{\partial r} \right)_{r=\rho} \quad (1.5)$$

Inserting (1.3) and (1.4) into (1.2) we obtain the condition  $n < 1$ .

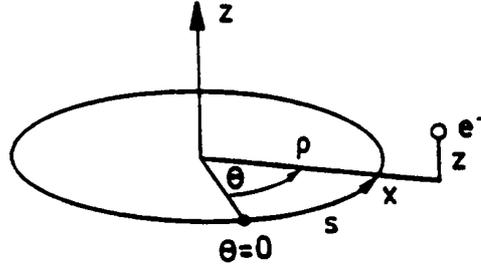


Figure 2: Circular design orbit and coordinates used to describe particle trajectories

It is worth noting that Eq. (1.2) is satisfied even with  $n \equiv 0$ , that is with a homogeneous field. This reflects the fact, that in a homogeneous field all plane orbits are circles, so that particles diverging from one point will meet again after  $180^\circ$  of revolution (see Fig. 3).

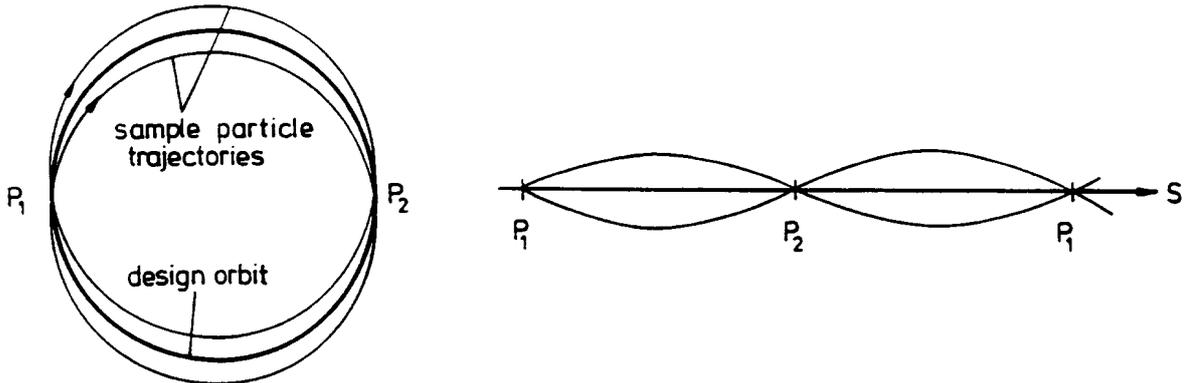


Figure 3: Geometrical focusing in a homogeneous magnetic field. All particles starting at  $P_1$  with slightly different angles  $\alpha$  move on circles with the same radii and meet again at  $P_2$ . With respect to the design orbit, this looks like focusing, which can be seen more clearly from the right-hand part of the figure where the design orbit has been developed into a straight line. The maximum deviation from the design orbit (i.e. the amplitude of betatron oscillations) is easily estimated at  $\alpha \cdot \rho$ . Thus for a typical beam divergence of  $\alpha = 1$  mrad it will be 1 mm at  $\rho = 1$  m but as large as 1000 mm in a high energy ring with  $\rho = 1000$  m.

This is called geometrical focusing, and it is an essential contribution to the weak focusing in the horizontal plane. It is, however, not sufficient:

Consider a particle in a homogeneous field with initial conditions such that it moves on a circle in the horizontal plane  $z \equiv 0$ . If it now gets an infinitely small kick into the vertical direction, it will spiral away to infinity and will be lost. To achieve stability in the vertical plane we need a restoring force

$$F_z = -C \cdot z \quad (C = \text{const.})$$

and therefore a horizontal field component