Pierre Henry-Labordère

Analysis, Geometry, and Modeling in Finance

Advanced Methods in Option Pricing

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To Emma and Véronique

List of Tables

2.1 Example of one-factor short-rate models. 47

5.1 Example of separable LV models satisfying $C(0) = 0$. 124
5.2 Feller criteria for the CEV model. 130

6.1 Example of SVMs. 151
6.2 Example of metrics for SVMs. 155

8.1 Example of stochastic (or local) volatility Libor market models. 208
8.2 Libor volatility triangle. 214

9.1 Feller boundary classification for one-dimensional Itô processes. 267
9.2 Condition at $z = 0$. 276
9.3 Condition at $z = 1$. 276
9.4 Condition at $z = \infty$ 278
9.5 Example of solvable superpotentials 278
9.6 Example of solvable one-factor short-rate models. 279
9.7 Example of Gauge free stochastic volatility models. 281
9.8 Stochastic volatility models and potential $J(s)$. 284

10.1 Example of potentials associated to LV models. 295

11.1 A dictionary from Malliavin calculus to QFT. 310

B.1 Associativity diagram. 360
B.2 Co-associativity diagram. 360

List of Figures

1.1 Implied volatility (multiplied by $\times 100$) for EuroStoxx50 (03-09-2007). The two axes represent the strikes and the maturity dates. Spot $S_0 = 4296$. 2

4.1 Manifold. 80
4.2 2-sphere. 82
4.3 Line bundle. 89

5.1 Comparison of the asymptotic solution at the first-order (resp. second-order) against the exact solution (5.23). $f_0 = 1$, $\sigma = 0.3$, $\beta = 0.33$, $\tau = 10$ years. 133
5.2 Comparison of the asymptotic solution at the first-order (resp. second-order) against the exact solution (5.23). $f_0 = 1$, $\sigma = 0.3$, $\beta = 0.6$, $\tau = 10$ years. 134
5.3 Market implied volatility (SP500, 3-March-2008) versus Dupire local volatility (multiplied by $\times 100$). $T = 1$ year. Note that the local skew is twice the implied volatility skew. 137
5.4 Comparison of the asymptotic implied volatility (5.41) at the zero-order (resp. first-order) against the exact solution (5.42). $f_0 = 1$, $\sigma = 0.3$, $\tau = 10$ years, $\beta = 0.33$. 142
5.5 Comparison of the asymptotic implied volatility (5.41) at the zero-order (resp. first-order) against the exact solution (5.42). $f_0 = 1$, $\sigma = 0.3\%$, $\tau = 10$ years, $\beta = 0.6$. 143

6.1 Poincaré disk $\mathcal{D}$ and upper half-plane $\mathbb{H}^2$ with some geodesics. In the upper half-plane, the geodesics correspond to vertical lines and to semi-circles centered on the horizon $\Im(z) = 0$ and in $\mathcal{D}$ the geodesics are circles orthogonal to $\mathcal{D}$. 169
6.2 Probability density $p(K,T|f_0) = \frac{\partial^2 \mathcal{C}(T,K)}{\partial^2 K}$. Asymptotic solution vs numerical solution (PDE solver). The Hagan-al formula has been plotted to see the impact of the mean-reverting term. Here f_0 is a swap spot and α has been fixed such that the Black volatility $\alpha f_0^{\beta-1} = 30\%$. 172
6.3 Implied volatility for the SABR model $\tau = 1$Y. $\alpha = 0.2$, $\rho = -0.7$, $\frac{\nu^2}{2}\tau = 0.5$ and $\beta = 1$. 174